

Mathematics for Machine Learning

Lab 1

Problem 1. Suppose $A = \begin{bmatrix} 2 & 6 \\ -1 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 5 & -3 & 7 \\ 1 & 4 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 6 & 5 \\ 4 & 3 \\ 2 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \\ 3 & 2 & 1 \end{bmatrix}$,

$E = [1 \ 6]$, $F = \begin{bmatrix} -7 \\ 3 \end{bmatrix}$. Compute the following matrices.

- | | |
|----------------|-------------------|
| 1) AF | 5) D^2 |
| 2) $A + BC$ | 6) EF |
| 3) CC^T | 7) $BC - E^T F^T$ |
| 4) $F^T A E^T$ | 8) $D(CB)$ |

Problem 2. Reduce the following matrices to row echelon form.

1) $\begin{bmatrix} 0 & 2 & 1 \\ 0 & 0 & 0 \\ 2 & 2 & 4 \end{bmatrix}$

3) $\begin{bmatrix} 1 & -3 & 3 & -4 \\ 2 & 3 & -1 & 15 \\ 4 & -3 & -1 & 19 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 2 & 1 & 3 \\ 4 & 2 & 0 & 0 \\ -1 & 3 & 2 & 1 \\ 1 & 2 & 5 & 2 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 & -1 & 1 & 6 \\ -1 & 1 & 2 & -1 & 3 \\ 2 & -1 & 2 & 2 & 14 \\ 1 & 1 & -1 & 2 & 8 \end{bmatrix}$

Problem 3. Solve the following systems of equations using Gaussian elimination for the odd numbered systems and Gauss-Jordan elimination for the even ones.

1) $\begin{cases} x_1 + x_2 = 5 \\ -3x_1 + 4x_2 = 13 \end{cases}$

3) $\begin{cases} 4x_2 + 6x_3 = 16 \\ 2x_1 + 3x_2 + x_3 = 5 \\ 4x_1 - 4x_2 - 8x_3 = -20 \end{cases}$

2) $\begin{cases} 2x_1 - 4x_2 + x_3 = -3 \\ 4x_1 + x_2 + 5x_3 = 21 \\ -5x_1 + 5x_2 - x_3 = 2 \end{cases}$

4) $\begin{cases} x_1 - x_2 - x_3 + 2x_4 = 1 \\ 2x_1 - 2x_2 - x_3 + x_4 = 3 \\ -x_1 + x_2 - x_3 = -3 \end{cases}$

Problem 4. Use the Gauss-Jordan method to compute the inverses of the following matrices if possible

1)
$$\begin{bmatrix} 3 & -2 & -1 \\ 0 & 0 & 1 \\ -1 & 1 & 0 \end{bmatrix}$$

2)
$$\begin{bmatrix} 2 & 4 & 5 \\ 0 & 0 & 1 \\ 0 & 0 & -3 \end{bmatrix}$$

3)
$$\begin{bmatrix} 2 & 2 & 3 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

4)
$$\begin{bmatrix} 1 & 2 & 3 \\ 5 & 1 & 1 \\ 3 & 6 & 9 \end{bmatrix}$$

5)
$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 0 & -3 \\ 2 & 0 & 2 \end{bmatrix}$$

6)
$$\begin{bmatrix} 2 & 1 & -4 \\ -4 & -1 & 6 \\ -1 & 1 & -1 \end{bmatrix}$$