Mathematics for Machine Learning

Lab 2

Problem 1. Compute the determinant of the following matrices.

1) $\begin{bmatrix} 1 & 2 & -2 \\ 2 & 0 & 1 \\ 1 & 1 & 3 \end{bmatrix}$

4) $\begin{bmatrix} 1 & 2 & 1 \\ -2 & 0 & -3 \\ 2 & 0 & 2 \end{bmatrix}$

2) $\begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$

5) $\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & -1 & 2 & 0 \\ 2 & -1 & 3 & 1 \\ 4 & 17 & 0 & -5 \end{bmatrix}$

3) $\begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 7 \\ 5 & 2 & 4 \end{bmatrix}$

6) $\begin{bmatrix} -2 & -4 & 7 \\ -3 & -6 & 10 \\ 1 & 2 & -3 \end{bmatrix}$

Problem 2. Show that the vectors **x, y, z** are linearly dependent in $\mathbb{R}^3$ and find
$\alpha, \beta \in \mathbb{R}$ s.t. $\alpha\mathbf{x} + \beta\mathbf{y} + \mathbf{z} = 0$.

1) $\mathbf{x} = (1, 1, 0)$, $\mathbf{y} = (0, 1, 2)$ and $\mathbf{z} = (3, 1, -4)$

2) $\mathbf{x} = (1, 2, -1)$, $\mathbf{y} = (3, 1, 1)$ and $\mathbf{z} = (5, -5, 7)$

Problem 3. Find all values of λ which make $\{\mathbf{u, v, w}\}$ a linearly dependent subset of
$\mathbb{R}^3$.

1) $\mathbf{u} = (\lambda, 1, 0)$, $\mathbf{v} = (1, \lambda, 1)$ and $\mathbf{w} = (0, 1, \lambda)$

2) $\mathbf{u} = (1, 0, -2)$, $\mathbf{v} = (1, 2, \lambda)$ and $\mathbf{w} = (2, 1, -1)$

Problem 4. Suppose the vectors $b_1, b_2, b_3, b_4 \in \mathbb{R}^n$ are linearly independent and

$$x_1 = 2b_1 + 3b_2 - b_3 - 3b_4$$
$$x_2 = 2b_1 - 4b_2 + 2b_3 + 8b_4$$
$$x_3 = 4b_1 + 2b_2 + b_4$$
$$x_4 = b_1 + 22b_2 - 5b_3 + 17b_4$$

Check whether the vectors $x_1, x_2, x_3, x_4 \in \mathbb{R}^n$ are linearly independent

Problem 5. Let the subspace $U = span(\boldsymbol{x_1}, \boldsymbol{x_2}, \boldsymbol{x_3}, \boldsymbol{x_4}) \subset \mathbb{R}^4$, where

$$x_1 = \begin{bmatrix} 2 \\ -1 \\ 1 \\ 2 \end{bmatrix} \qquad x_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ -1 \end{bmatrix} \qquad x_3 = \begin{bmatrix} 1 \\ -8 \\ 5 \\ 6 \end{bmatrix} \qquad x_4 = \begin{bmatrix} -3 \\ 4 \\ -3 \\ 5 \end{bmatrix}$$

Find out which vectors are a basis for U.

Problem 6. Compute the rank of the following matrices.

1) $\begin{bmatrix} 0 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 2 & 4 \end{bmatrix}$
3) $\begin{bmatrix} 4 & 0 & 2 & 3 \\ 5 & 1 & 0 & 1 \\ 1 & 2 & 4 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 2 & 1 & 3 \\ 4 & 2 & 0 & 0 \\ -1 & 3 & 2 & 1 \\ 1 & 2 & 5 & 2 \end{bmatrix}$
4) $\begin{bmatrix} 2 & -4 & -2 & 6 \\ 3 & 1 & 6 & 6 \\ 1 & 2 & 3 & 3 \\ 3 & 2 & 1 & 4 \end{bmatrix}$

Problem 7. Given the ordered basis

$$B = \begin{bmatrix} 1 & -1 & 3 \\ -1 & 4 & 9 \\ 3 & 9 & 4 \end{bmatrix},$$

find the coordinate vector of $\boldsymbol{x} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$.