

Mathematics for Machine Learning

Lab 4

Problem 1. What is the geometric and algebraic multiplicity of the eigenvalues of the given matrices?

1) $\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$

2) $\begin{bmatrix} 2 & 0 & 3 \\ 0 & 3 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

Problem 2. Compute

- the eigenvalues, trace, determinant and the characteristic polynomial of the matrix A
- the characteristic polynomial of the matrix A^T ,

given that

$$A\mathbf{x}_1 = 2\mathbf{x}_1, A\mathbf{x}_2 = 3\mathbf{x}_2, A\mathbf{x}_3 = -4\mathbf{x}_3,$$

where $A \in \mathbb{R}^{3 \times 3}$ and $\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3 \in \mathbb{R}^3$.

Problem 3. Find the lower-triangular matrix L (Cholesky factor) such that $A = LL^T$ if A is

1) $A = \begin{bmatrix} 1 & 2 \\ 2 & 6 \end{bmatrix}$

2) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 12 \\ 3 & 12 & 27 \end{bmatrix}$

Problem 4. Factorize the following matrices using SVD

1) $A = \begin{bmatrix} 3 & 0 \\ 4 & 5 \end{bmatrix}$

2) $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

Problem 5. Use the definition of limit to prove the following

1) $\lim_{n \rightarrow \infty} \frac{n}{n+1} = 1$

3) $\lim_{n \rightarrow \infty} \frac{4n \cos n}{n^2 - n + 6} = 0$

2) $\lim_{n \rightarrow \infty} \frac{(-1)^n + 3}{2n} = 0$

4) $\lim_{n \rightarrow \infty} \frac{\sqrt{n^2 + 1}}{\sqrt{n^2 + 2n - 1}} = 1$