

Mathematics for Machine Learning

Lab 7

Problem 1. Calculate the following limits

$$\begin{array}{ll}
 \text{1)} \quad \lim_{(x,y) \rightarrow (2,3)} \frac{\sqrt{3x} - \sqrt{2y}}{3x - 2y} & \text{3)} \quad \lim_{(x,y) \rightarrow (0,0)} \frac{\sin xy}{xy} \\
 \text{2)} \quad \lim_{(x,y) \rightarrow (0,0)} (x+y) \sin \frac{1}{x^2 + y^2} & \text{4)} \quad \lim_{(x,y) \rightarrow (1,1)} \frac{2x^2 - xy - y^2}{x^2 - y^2}
 \end{array}$$

Problem 2. Compute the directional derivative $D_{\mathbf{v}}f(x_0, y_0)$ if

- 1) $f(x, y) = x^2y$, $(x_0, y_0) = (1, 2)$, $\mathbf{v} = (1, 1)$
- 2) $f(x, y) = x^3 - 3xy + 4y^2$, $(x_0, y_0) = (2, 1)$,
 \mathbf{v} is the vector that forms an angle $\frac{\pi}{3}$ with the x-axis

Problem 3. Find the extremum points of these functions.

- 1) $f(x, y) = x^2 - xy + y^2 - 2x + y$
- 2) $f(x, y) = x^3 + y^3 - 3xy$
- 3) $f(x, y) = x^4 + y^4 - x^2 - 2xy - y^2$
- 4) $f(x, y) = \sin x + \cos x + \cos(x - y)$, where $x, y \in [0, \frac{\pi}{2}]$

Problem 4. Suppose we roll two fair dice. What is the probability of getting

- 1) 5 on each of them, given that the sum of the resulting numbers is divisible by 5.
- 2) at least one 6, given that the sum of the two numbers is 8.

Problem 5. There are two boxes containing $\{5, 11, 8\}$ and $\{10, 8, 6\}$ white, black, red balls respectively. One ball is drawn from each box. What is the probability that the balls have the same color?

Problem 6. There are 2 white, 3 black and 5 red balls in a box. If we randomly take out 3 balls, what is the probability that at least 2 of them have different colors?