Deep Learning

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FAST DISCOVERING THE FUTURE

Outline

KL Divergence

The KL divergence (also called relative entropy) is a measure of how one probability distribution is different from a second, reference probability distribution:

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but there is no symmetry, i.e. $KL(P||Q) \neq KL(Q||P)$.

Jensen-Shannon Divergence

JS Divergence is the following

$$JS\left(P||Q\right) = \frac{1}{2}KL\left(P||M\right) + \frac{1}{2}KL\left(M||Q\right),$$

where $M = \frac{P+Q}{2}$.

KL Divergence for Gaussians

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Recall that probability density function (if it exists) of multivariate normal distribution with mean μ and with (non-singular, symmetric, positive definite) covariance matrix Σ is the following function:

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Suppose that we have two multivariate normal distributions: $\mathcal{N}_1(\mu_1, \Sigma_1), \mathcal{N}_2(\mu_2, \Sigma_2)$. Then

$$KL(\mathcal{N}_1, \mathcal{N}_2) = \frac{1}{2}\left(\mathrm{tr}\left(\Sigma_2^{-1}\Sigma_1\right) + (\mu_2-\mu_1)^{\mathsf{T}}\Sigma_2^{-1}(\mu_2-\mu_1) - k + \ln\frac{|\Sigma_2|}{|\Sigma_1|}\right).$$

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In one dimensional case we will have

$$KL(\mathcal{N}_1, \mathcal{N}_2) = \log\frac{\sigma_2}{\sigma_1} + \frac{\sigma_1^2 + (\mu_1-\mu_2)^2}{2\sigma_2^2} - \frac{1}{2}.$$

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Examples?

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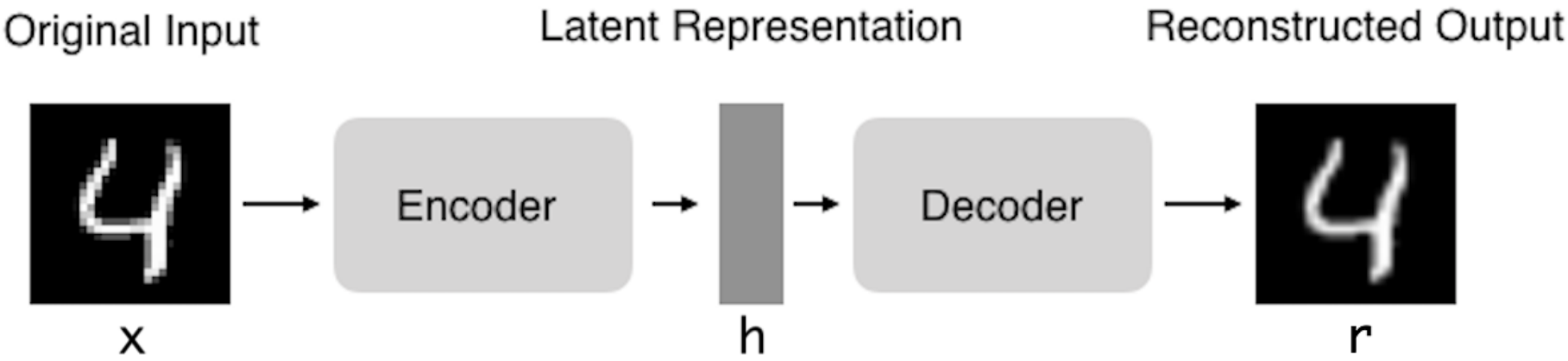
- **Decoder**:
  This part aims to reconstruct the input from the latent space representation.

Autoencoders



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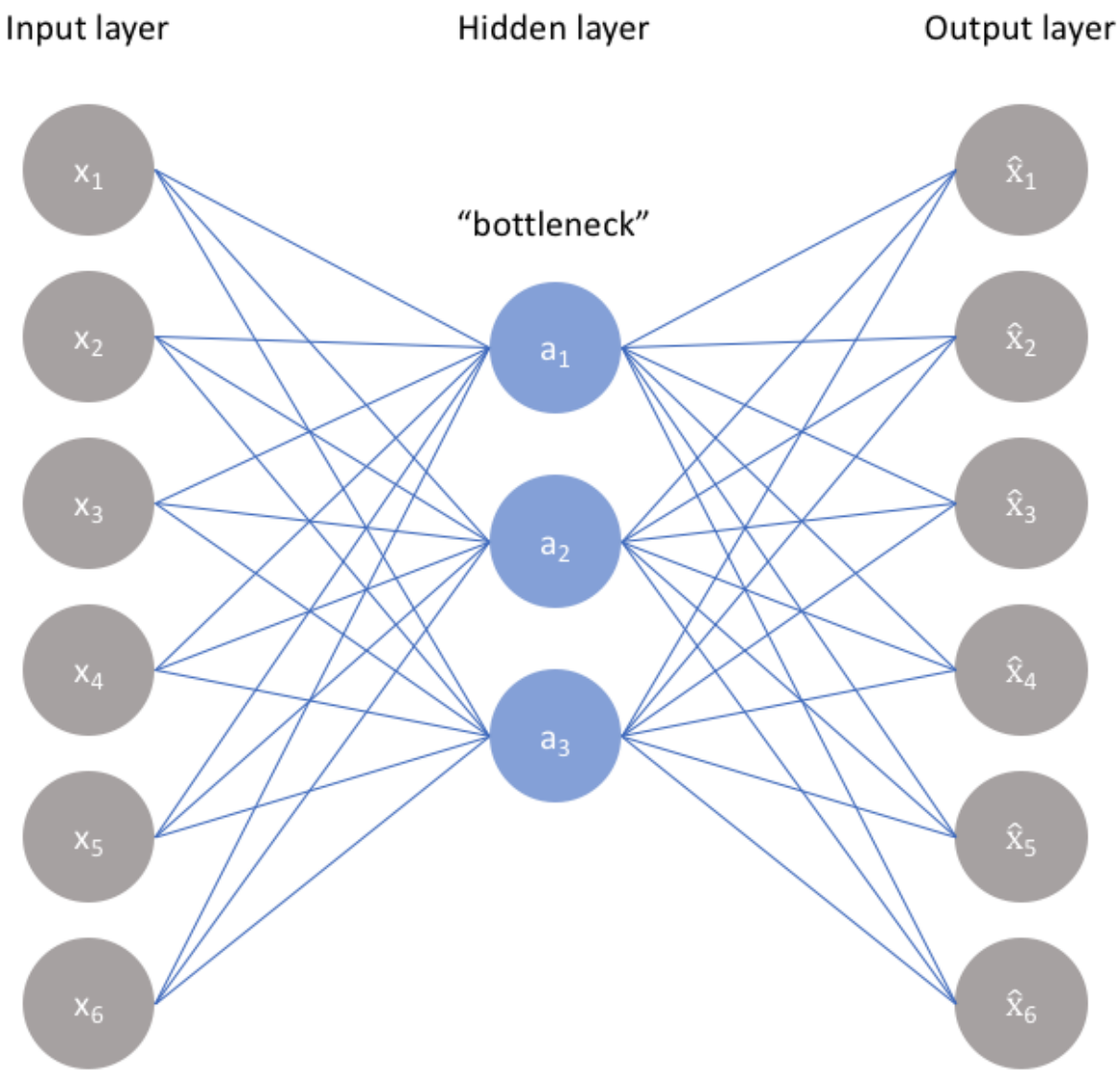
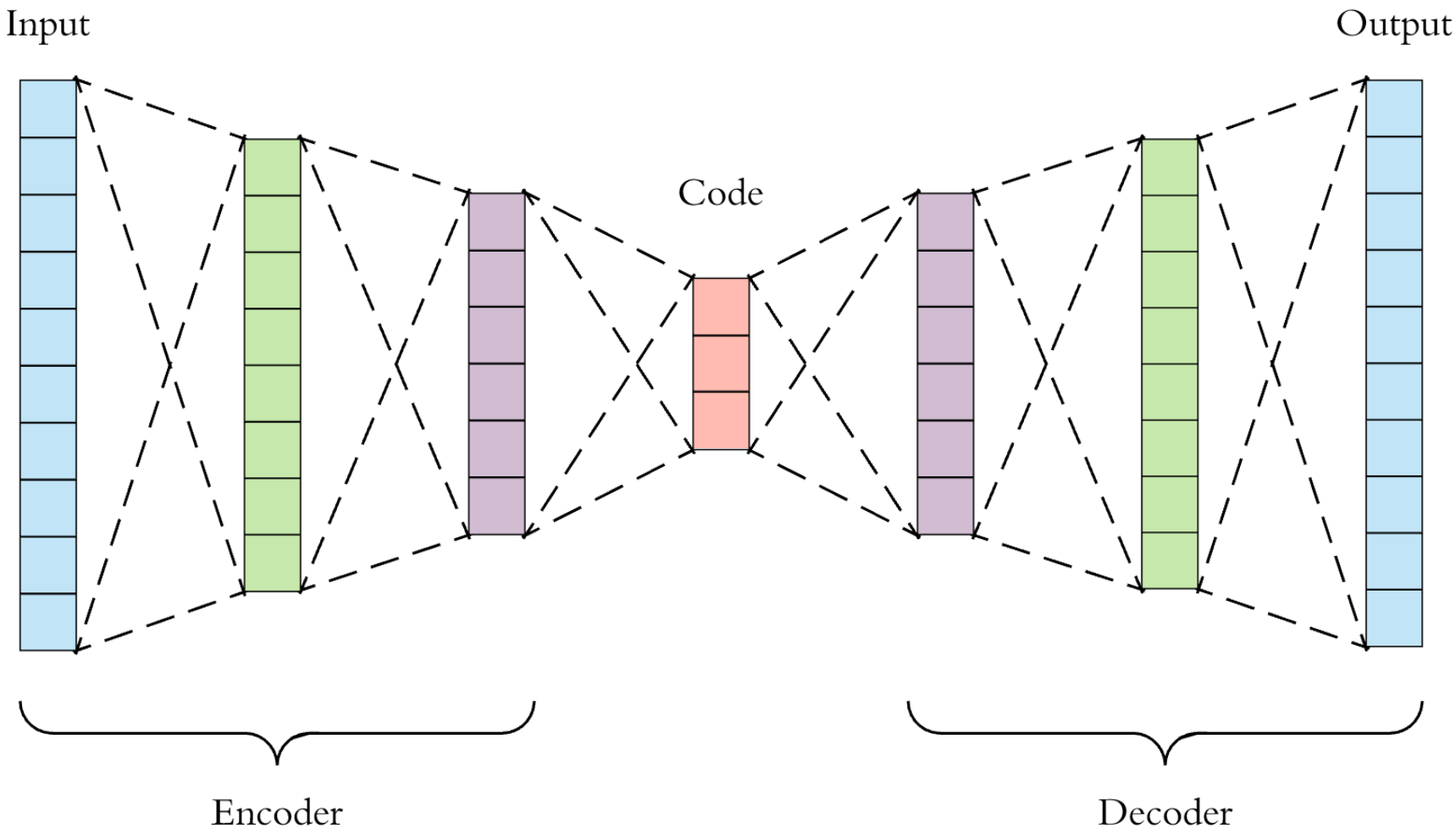
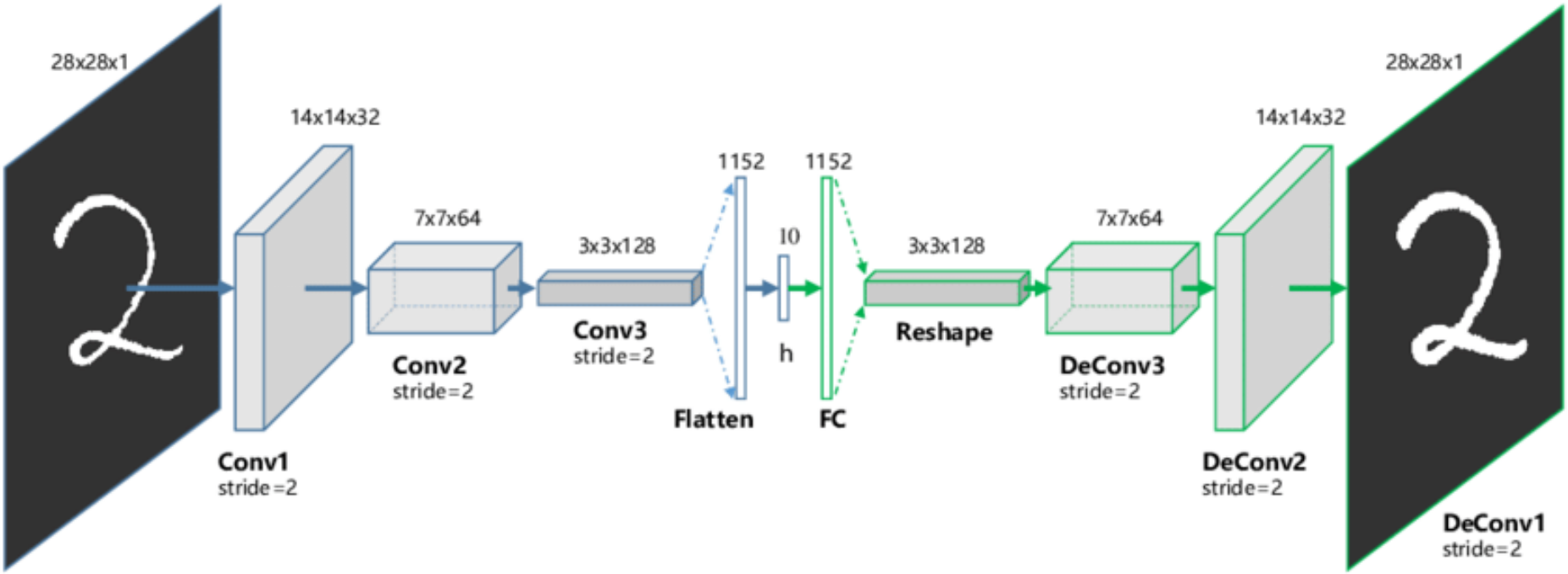
Vanilla Autoencoders



Multilayer/Deep Autoencoders



Convolutional Autoencoders



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- This is accomplished by adding a regularizer, or penalty term, to whatever cost or objective function the algorithm is trying to minimize.
- The end result is to reduce the learned representation's sensitivity towards the training input.

Contractive Autoencoders

Let f is our encoder, g is the decoder and D is our training dataset. In the previous cases we minimize this kind of loss function:

$$\sum_{x \in D} L\left(x, g\left(f\left(x\right)\right)\right).$$

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In the case of contractive autoencoders we will minimize this one

$$\sum_{x \in D} \left(L\left(x, g\left(f\left(x\right)\right)\right) + \lambda \left\| J_f\left(x\right) \right\|_F^2 \right),$$

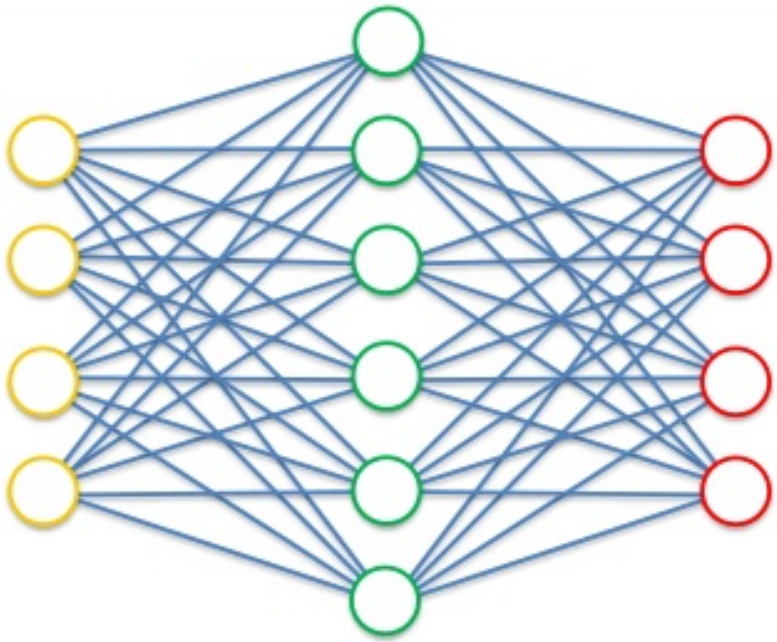
where the added summand is the square of Frobenius norm of the following Jacobian matrix:

$$\left[J_f\left(x\right)\right]_{i,j} = \frac{\partial f_j\left(x\right)}{\partial x_i}$$

i.e.

$$\left\| J_f\left(x\right) \right\|_F^2 = \sum_{i,j} \left(\frac{\partial f_j\left(x\right)}{\partial x_i} \right)^2.$$

Sparse Autoencoders



Deep Learning A-Z

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- Sparse autoencoders have hidden nodes greater than input nodes. They can still discover important features from the data.
- Sparsity penalty is introduced on the hidden layer. This is to prevent output layer copy input data. This prevents overfitting.

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Let f is our encoder, g is the decoder and D is our training dataset, which has n samples. Denote

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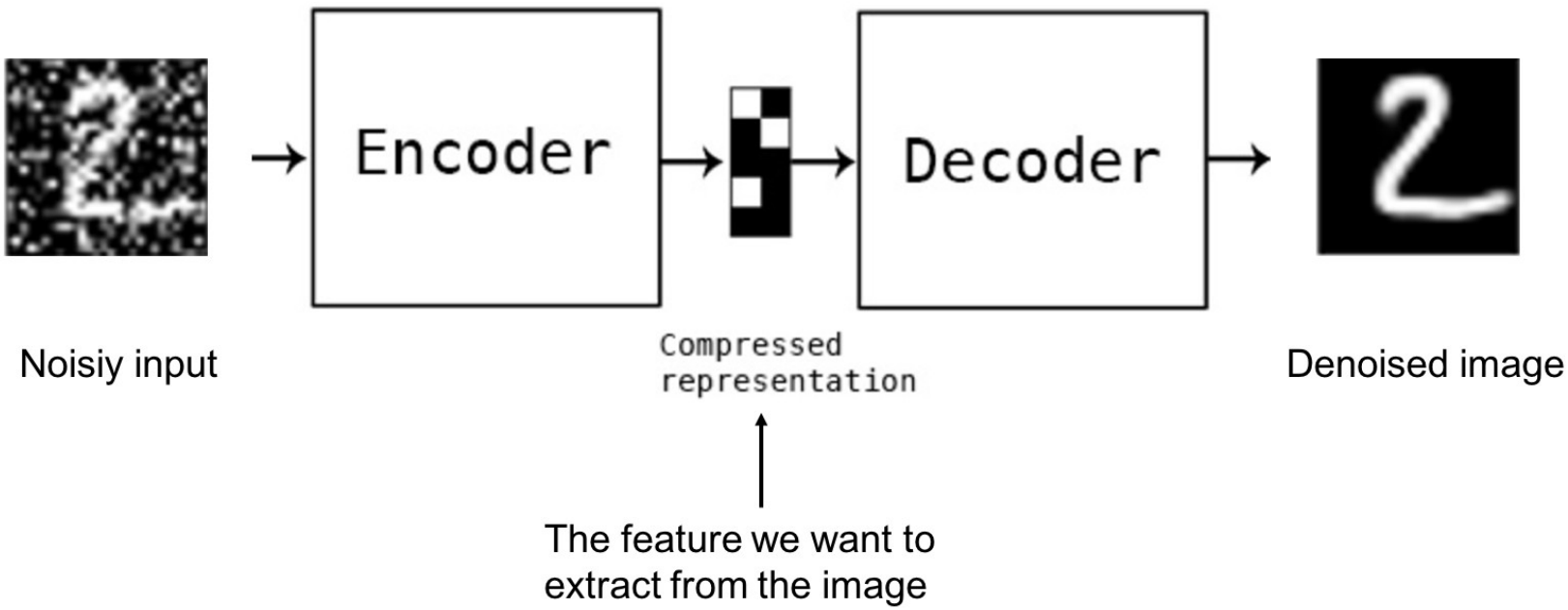
$$KL(\rho || \rho_j) = -\rho \log \frac{\rho_j}{\rho} - (1 - \rho) \log \frac{1 - \rho_j}{1 - \rho}.$$

Denoising Autoencoders



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- The model learns a vector field for mapping the input data towards a lower dimensional manifold which describes the natural data to cancel out the added noise.