

Mathematics for Machine Learning

Lab 8

Problem 1. Suppose that you know the answers to 20 questions out of the total 25 questions in the entrance examination for the Unit 1991, from which you are only asked 3 random questions. What is the probability that you answer to all of the questions?

Problem 2. There are two boxes containing $\{5, 11\}$ and $\{10, 8\}$ white, black balls respectively. Suppose we take out a ball from each of the boxes and then randomly choose one of them. What is the probability that the final ball is white?

Problem 3. There are three boxes having 4 white and 6 black balls in each of them. Suppose we perform the following steps in the given order:

1. take out a ball from the first box and put it into the second one
2. take out a ball from the second box and put it into the third one
3. take out a ball from the third box and put it into the first one

What is the probability of taking a white ball from the third box?

Problem 4. Two factories produce similar weapons and deliver it to the army warehouse. The first factory's productivity is two times more than that of the second one. Moreover, 40% of the weapons produced by the first factory have some defects, while the same indicator is only 16% for the second factory. We randomly take a weapon from the warehouse, test it and it appears to have no defects. What is the probability that the weapon was produced by the first factory?

Problem 5. According to some statistics, 5% of all males and 0.25% of all females are color-blind. Assuming that the population of males and females are equal, what is the probability that a randomly chosen person is a color-blind man?

Problem 6. Suppose that 3 soldiers can hit the target with one shot with probabilities 0.8, 0.75 and 0.6. Once, they shot all together at the same time and one of them missed the target. What is the probability that it was the third soldier?

- Problem 7.** Suppose we roll a die 10 times in a row. What is the probability of getting a number divisible by 3 two times?
- Problem 8.** Suppose we perform 5 independent tests, during which we throw 3 dice at the same time. What is the probability of getting all 5s during 4 tests.
- Problem 9.** The probability of scoring at least one goal from the penalty spot is 0.96 for some football player, who did 4 attempts in total. What is the probability of scoring a goal from penalty for that player, if we assume that this probability is unchanged during every attempt?
- Problem 10.** Suppose we shoot at the target until the first hit on the target. What is the probability of shooting exactly 8 times, if the probability of hitting the target with one shot is 0.8?
- Problem 11.** A coin is tossed twice. Write out the distribution rule and the probability mass function of the number of times Heads appears.
- Problem 12.** 10% of some products is defected. What will be the distribution rule of the number of defected products if we take 4 of them randomly?
- Problem 13.** Write out the distribution rule of the number of total shots described in *Problem 10* above.
- Problem 14.** The distribution function of X is given by

$$F_X(x) = \begin{cases} 0, & \text{if } x \leq 0 \\ x^2, & \text{if } x \in (0, 1] \\ 1, & \text{if } x > 1 \end{cases}.$$

What is the probability of getting a value from $[0.2, 0.8]$ in three out of four independent trials?