Deep Learning

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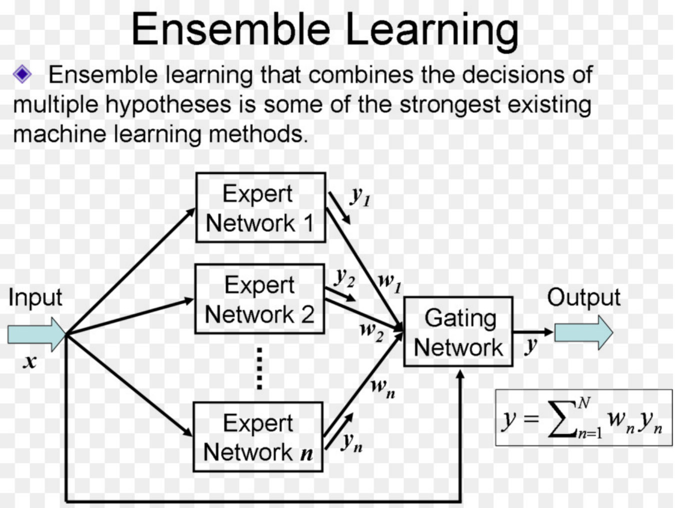
FAST DISCOVERING THE FUTURE

Outline

Ensemble of Neural Networks



Ensemble of Neural Networks

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Can we do ensemble learning with infinite number of neural networks?

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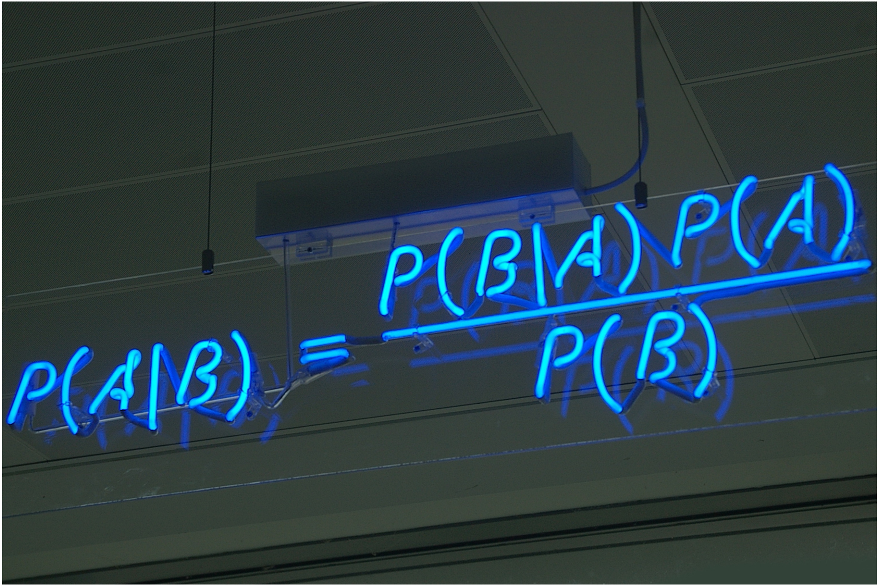
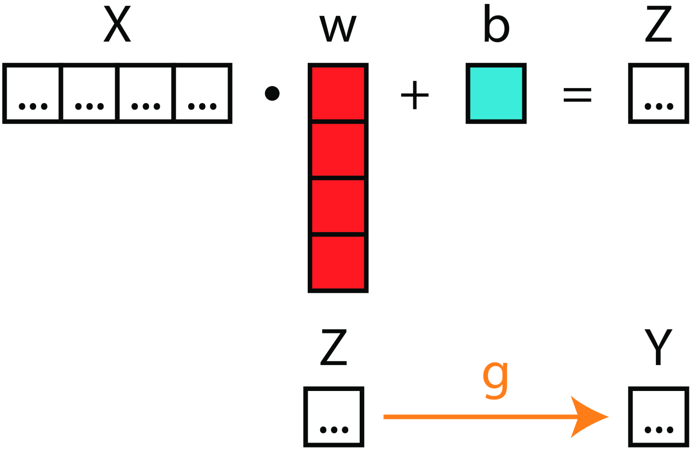
BNNs




X
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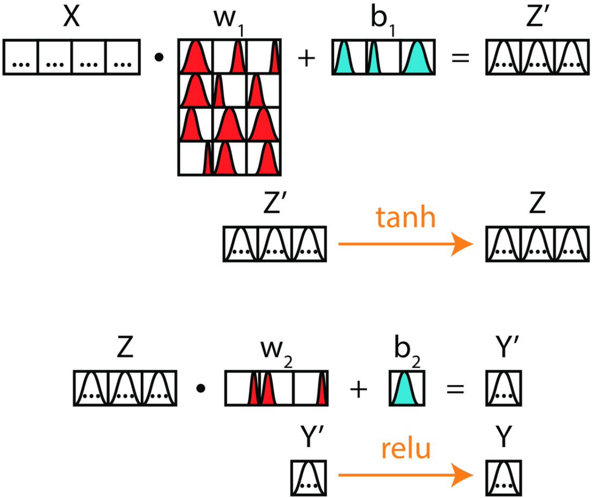
BNNs



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$$w^{MAP} = \underset{w}{\operatorname{argmax}}\ p\left(w|\mathcal{D}\right) = \underset{w}{\operatorname{argmax}}\ \frac{p\left(\mathcal{D}|w\right) p\left(w\right)}{p\left(\mathcal{D}\right)}$$

$$= \underset{w}{\operatorname{argmax}} \left(\log p\left(\mathcal{D}|w\right) + \log p\left(w\right)\right).$$

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We will assume that prior $p(w)$ is mixture of two Gaussians:

$$p\left(w\right) = \prod_j \left(\alpha\mathcal{N}\left(w_j|0,\sigma_1^2\right) + \left(1-\alpha\right)\mathcal{N}\left(w_j|0,\sigma_2^2\right)\right)$$

where the first mixture component of the prior is given a larger variance than the second: $\sigma_1 > \sigma_2$.

BNNs

1. Sample $\epsilon \sim \mathcal{N}(0, I)$.
2. Let $\mathbf{w} = \mu + \log(1 + \exp(\rho)) \circ \epsilon$.
3. Let $\theta = (\mu, \rho)$.
4. Let $f(\mathbf{w}, \theta) = \log q(\mathbf{w}|\theta) - \log P(\mathbf{w})P(\mathcal{D}|\mathbf{w})$.
5. Calculate the gradient with respect to the mean

$$\Delta_\mu = \frac{\partial f(\mathbf{w}, \theta)}{\partial \mathbf{w}} + \frac{\partial f(\mathbf{w}, \theta)}{\partial \mu}. \tag{3}$$

6. Calculate the gradient with respect to the standard deviation parameter ρ

$$\Delta_\rho = \frac{\partial f(\mathbf{w}, \theta)}{\partial \mathbf{w}} \frac{\epsilon}{1 + \exp(-\rho)} + \frac{\partial f(\mathbf{w}, \theta)}{\partial \rho}. \tag{4}$$

7. Update the variational parameters:

$$\mu \leftarrow \mu - \alpha \Delta_\mu \tag{5}$$

$$\rho \leftarrow \rho - \alpha \Delta_\rho. \tag{6}$$

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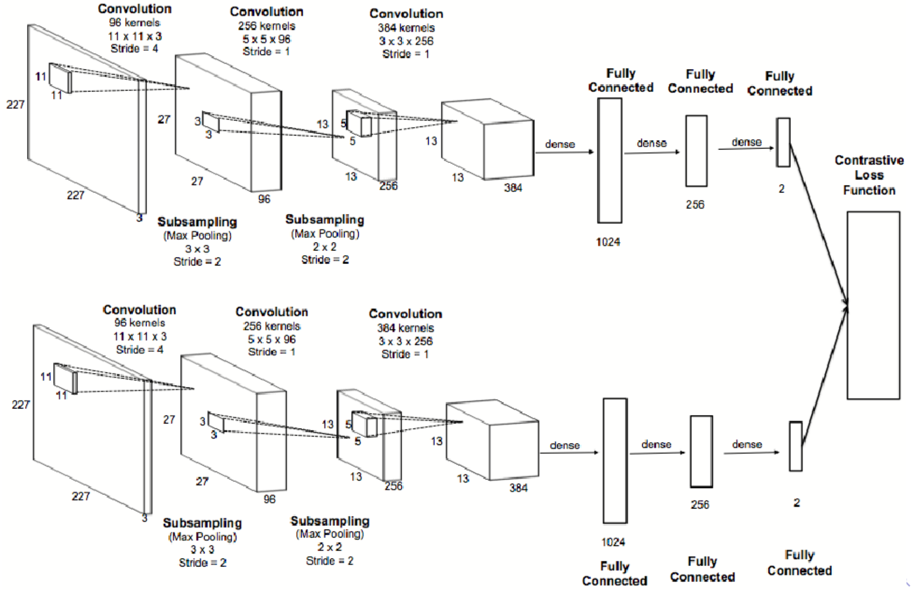
- What to do?

Siamese NN


Convolution
96 kernels
11 x 11 x 3
Stride = 4
Convolution
256 kernels
5 x 5 x 96
Stride = 1
Convolution
384 kernels
3 x 3 x 256
Stride = 1
227
11
11
227
3
27
3
3
27
96
13
5
5
13
256
13
13
384
Subsampling
(Max Pooling)
3 x 3
Stride = 2
Subsampling
(Max Pooling)
2 x 2
Stride = 2
dense
dense
dense
Fully Connected
Fully Connected
Fully Connected
1024
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2
Contrastive Loss Function
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Triplet Loss Function

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- p represents a random positive sample,
- n represents a negative sample,
- m is an arbitrary margin and is used to further the separation between the positive and negative scores,
- if $m = 0.2$ and $d\left(a, p\right) = 0.5$ then $d\left(a, n\right)$ should at least be equal to 0.7.