FAST DISCOVERING THE FUTURE

Mathematics for Machine Learning

Lab 11

Problem 1. Find the Fourier transform of the following functions

1) $f(t) = e^{-2|t|}$

2) $f(t) = \begin{cases} 1, \text{if } |t| \leq 1 \\ 0, \text{if } |t| > 1 \end{cases}$

Problem 2. Find the Fourier series representation of the function f and use it to show the given equalities

1) $f(x) = \begin{cases} \pi - x, \text{if } x \in (0, \pi) \\ 0, \text{if } x \in (\pi, 2\pi) \end{cases}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$

2) $f(x) = x$, where $x \in (-\pi, \pi)$ and $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k-1)} = \frac{\pi}{4}$

Problem 3. Find the DFT of the following sequence

$$x_n = 2\delta_n + 3\delta_{n-1} - \delta_{n-2} + \delta_{n-3}, \quad n = 0, 1, 2, 3$$

Compute the IDFT from the resulting DFT to check whether we get the initial sequence.