Deep Learning

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FAST DISCOVERING THE FUTURE

Outline

Stochastic Gradient Descent

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Do you see problems in finding minimum of these functions using GD?

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Do you see problems in this optimization method?

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  - For $i = 1, 2, ..., \lceil \frac{n}{B} \rceil$, do

$$w \leftarrow w - \alpha \nabla \frac{1}{B} \sum_{k=(i-1)\cdot B+1}^{i \cdot B} L_k(w).$$

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Back-Propagation

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Theorem 1

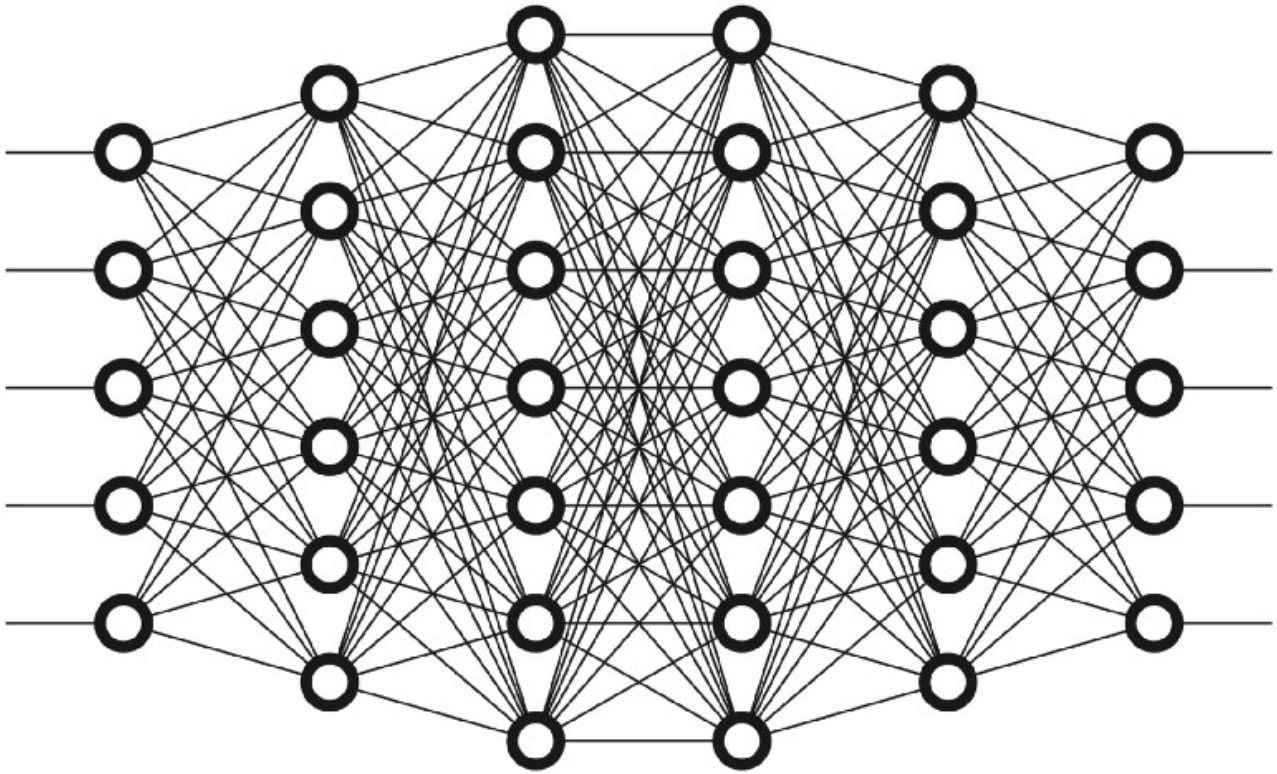
Given n functions $f_1, \ldots, f_n$ with the composite function

$$f = f_1 \circ (f_2 \circ \cdots (f_{n-1} \circ f_n)),$$

if each function f_i is differentiable at its immediate input, then the composite function is also differentiable by the repeated application of Chain Rule, where the derivative is

$$\frac{df}{dx} = \frac{df_1}{df_2}\frac{df_2}{df_3} \cdots \frac{df_n}{dx}.$$

Back-Propagation



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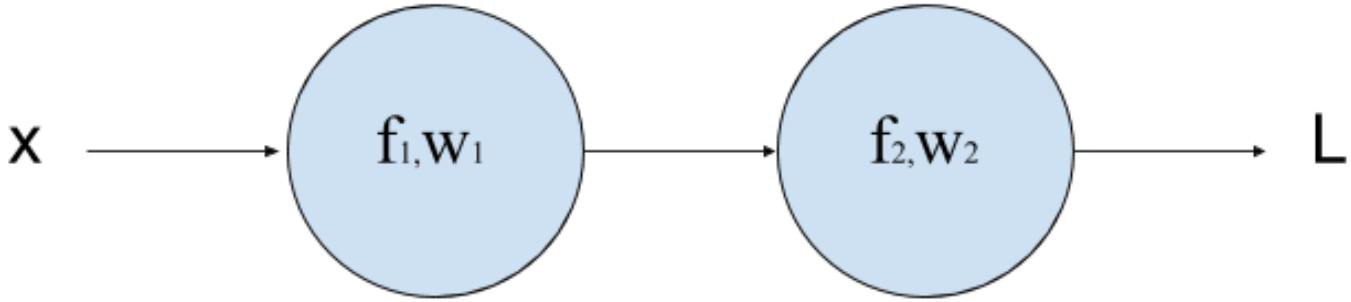

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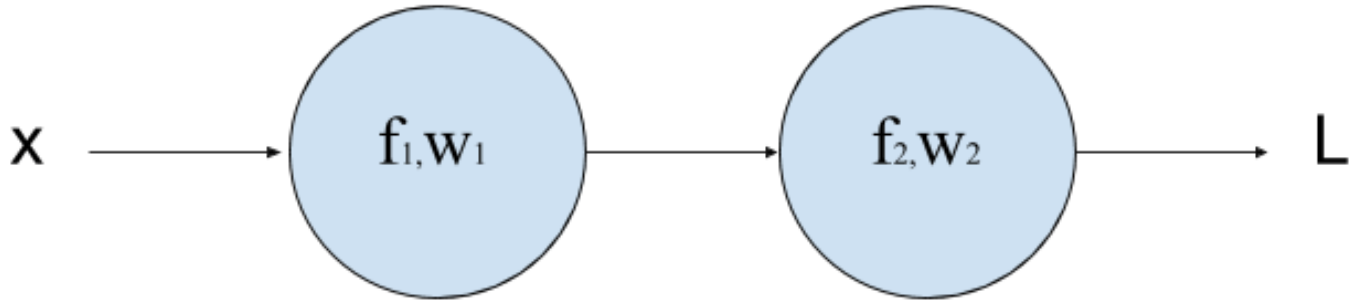

In this case we have the following function

$$L(w_1, w_2) = (f_2(w_2 f_1(w_1 x)) - y)^2$$

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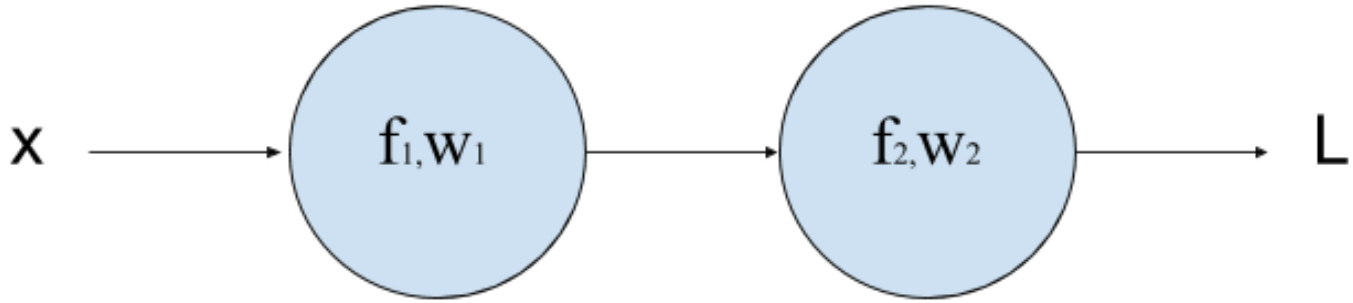

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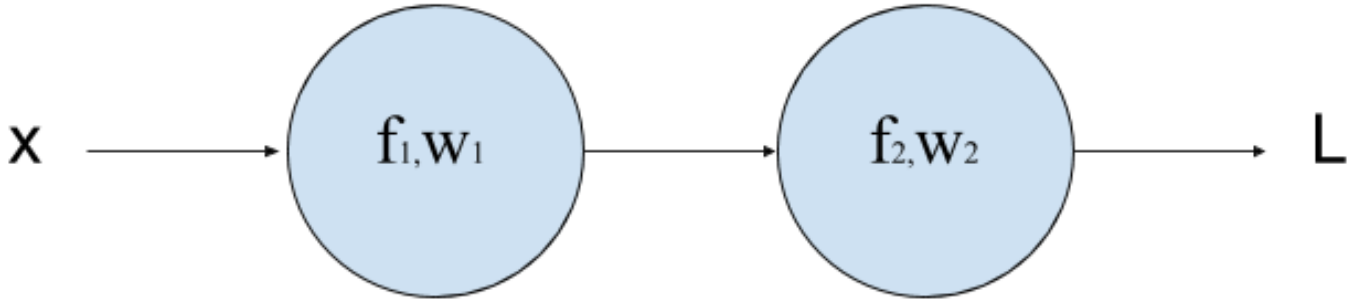

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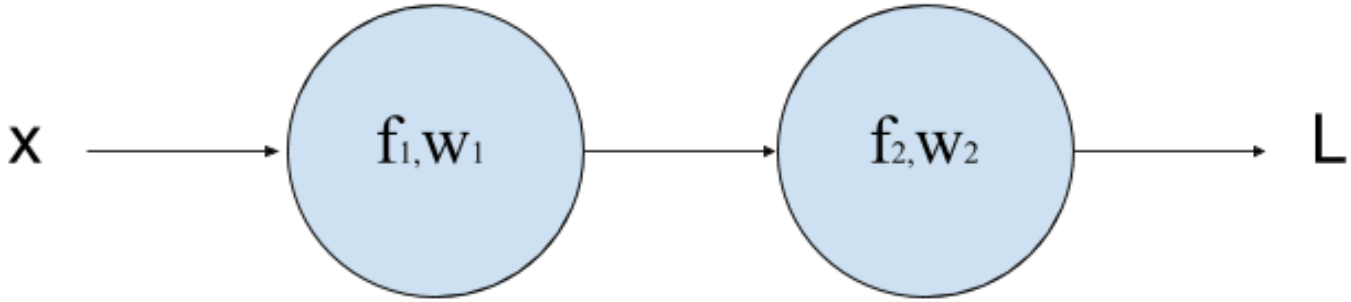

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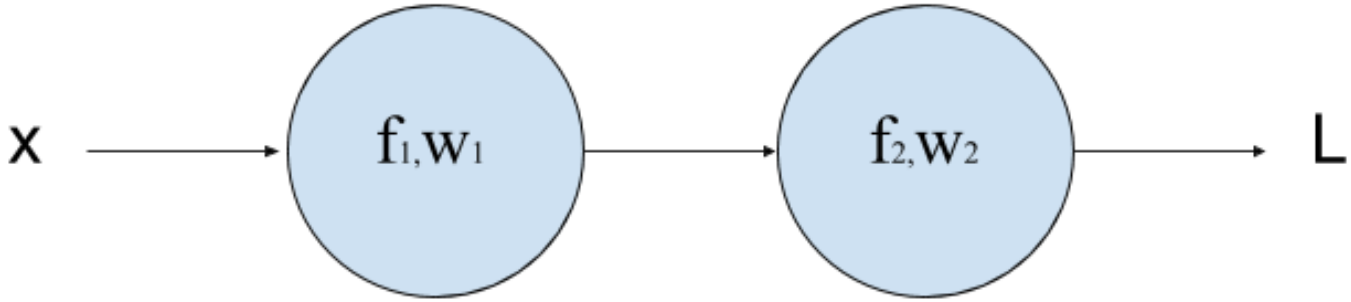

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