

Mathematics for Machine Learning: Homework 8

Deadline is 10.09.2020

September 1, 2020

1. A recent college graduate is planning to take the first three actuarial examinations in the coming summer. She will take the first actuarial exam in June. If she passes that exam, then she will take the second exam in July, and if she also passes that one, then she will take the third exam in September. If she fails an exam, then she is not allowed to take any others. The probability that she passes the first exam is 0.9. If she passes the first exam, then the probability that she passes the second one is 0.8, and if she passes both the first and the second exams, then the probability that she passes the third exam is 0.7.
 - (a) What is the probability that she passes all three exams?
 - (b) Given that she did not pass all three exams, what is the probability that she failed the second exam?
2. Ninety-eight percent of all babies survive delivery. However, 15 percent of all births involve Cesarean (C) sections, and when a C section is performed, the baby survives 96 percent of the time. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives?
3. In a certain community, 36 percent of the families own a dog and 22 percent of the families that own a dog also own a cat. In addition, 30 percent of the families own a cat. What is
 - (a) the probability that a randomly selected family owns both a dog and a cat?
 - (b) the probability that a randomly selected family owns a dog given that it owns a cat?
4. Three cooks, A, B, and C, bake a special kind of cake, and with respective probabilities 0.02, 0.03, and 0.05, it fails to rise. In the restaurant where they work, A bakes 50 percent of these cakes, B 30 percent, and C 20 percent. What proportion of "failures" is caused by A?
5. Urn A has 5 white and 7 black balls. Urn B has 3 white and 12 black balls. We flip a fair coin. If the outcome is heads, then a ball from urn A is selected, whereas if the outcome is tails, then a ball from urn B is selected. Suppose that a white ball is selected. What is the probability that the coin landed tails?

6. A simplified model for the movement of the price of a stock supposes that on each day the stock's price either moves up 1 unit with probability p or moves down 1 unit with probability $1 - p$. The changes on different days are assumed to be independent.
- (a) What is the probability that after 2 days the stock will be at its original price?
- (b) What is the probability that after 3 days the stock's price will have increased by 1 unit?
- (c) Given that after 3 days the stock's price has increased by 1 unit, what is the probability that it went up on the first day?
7. Suppose that each child born to a couple is equally likely to be a boy or a girl, independently of the sex distribution of the other children in the family. For a couple having 4 children, let X be the number of boys in their family. Find the PMF and CDF of X .
8. If the distribution function of X is given by

$$F(x) = \begin{cases} 0, & x < 1; \\ 0.2, & 1 \leq x < 3; \\ 0.4, & 3 \leq x < 3.5; \\ 0.9, & 3.5 \leq x < 5; \\ 1, & x \geq 5; \end{cases}$$

calculate the probability mass function of X .