Deep Learning

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October 24, 2020

FAST DISCOVERING THE FUTURE

Outline

1 Linear and Logistic Regressions

2 Softmax Classifier

Linear Regression

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We choose L^2 distance as our loss function:

$$\frac{1}{n} \sum_{l=1}^{n} \left(f(x_l) - y_l\right)^2.$$

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We choose cross entropy distance as our loss function:

$$\frac{1}{n} \sum_{l=1}^{n} \left(-y_l \log f(x_l) - (1 - y_l) \log\left(1 - f(x_l)\right)\right).$$

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4. Can we do logistic regression when number of classes is greater than 2?

L1 and L2 Regularizations

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In linear regression instead of L^2 loss we use one from this two:

$$\frac{1}{n}\sum_{l=1}^{n}\left(f\left(x_l\right)-y_l\right)^2+\frac{\lambda}{k}\sum_{l=1}^{k}\left|w_l\right|,$$

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3. What to do in the case of multi-label classification?