

Mathematics for Machine Learning

Lab 6

Problem 1. Compute the following integrals.

$$\begin{array}{ll}
 1) \int_1^2 x \sin(x^2) dx & 4) \int_0^1 (1-x^2)^{\frac{3}{2}} dx \\
 2) \int_0^\pi x \sin x dx & 5) \int_0^1 \frac{x+3}{(x+1)^2} dx \\
 3) \int_0^1 x e^{2x} dx & 6) \int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} t g x dx
 \end{array}$$

Problem 2. Compute the given improper integrals.

$$\begin{array}{ll}
 1) \int_1^\infty \frac{dx}{x^2} & 3) \int_1^\infty \frac{dx}{x^2 + x - 2} \\
 2) \int_1^\infty e^{-3x} dx & 4) \int_0^4 \frac{dx}{x + \sqrt{x}}
 \end{array}$$

Problem 3. Find the Taylor series for $f(x)$ centered at the given value of a .

$$\begin{array}{ll}
 1) f(x) = \sin^2 x, \quad a = 0 & 3) f(x) = x^2 e^x, \quad a = 0 \\
 2) f(x) = \frac{1}{x}, \quad a = -3 & 4) f(x) = \sqrt{x}, \quad a = 16
 \end{array}$$

Problem 4. Find the indicated partial derivatives.

$$\begin{array}{l}
 1) \frac{\partial z}{\partial s}, \frac{\partial z}{\partial t}, \frac{\partial z}{\partial u} \text{ when } s = 4, \quad t = 2, \quad u = 1 \\
 \quad z = x^4 + x^2 y, \quad x = s + 2t - u, \quad y = stu^2; \\
 2) \frac{\partial w}{\partial r}, \frac{\partial w}{\partial \theta}, \text{ when } r = 2, \quad \theta = \frac{\pi}{2} \\
 \quad w = xy + zy + zx, \quad x = r \cos \theta, \quad y = r \sin \theta, \quad z = r\theta \\
 3) \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \text{ when } x = 0, \quad y = 2 \\
 \quad f = u^2 + v^2 + w^2, \quad u = xe^y, \quad v = ye^x, \quad w = e^{xy}
 \end{array}$$

Problem 5. Calculate $\nabla f(a, b)$, if

$$a. f(x, y) = x^2 \ln y \quad (a, b) = (2, 3)$$

b. $f(x, y) = x^2y - xy \quad (a, b) = (2, -1)$

c. $f(x, y) = e^x \sin y \quad (a, b) = (0, \frac{\pi}{3})$

d. $f(x, y) = x^2e^{-y}, \quad (a, b) = (3, 0)$