

Mathematics for Machine Learning: Homework 9

Deadline is 17.09.2020

September 9, 2020

1. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $1/100$, what is the (approximate) probability that you will win a prize
 - (a) at least once?
 - (b) exactly once?
 - (c) at least twice?
2. The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that there will be
 - (a) at least 2 such accidents in the next month;
 - (b) at most 1 accident in the next month?
3. Let the range of the random variable X is $R = \{0, \pm 1, \pm 2, \dots, \pm n\}$ and $\mathbb{P}(X = i) = \frac{1}{2n+1}$ for all $i \in R$. Compute $E[X]$ and $Var(X)$.
4. Let X be a binomial random variable with parameters n, p . Compute $\mathbb{E}[X^3]$.
5. Let X be a Poisson random variable with parameter λ . Compute $\mathbb{E}[X^3]$.
6. Let X be a geometric random variable with parameter p . Compute $Var(X)$.
7. Find the domain of convergence of the functional sequence f_n and calculate its limit.

a) $f_n(x) = \sin^n x,$

b) $f_n(x) = \sqrt[n]{e^{-nx} + n^{10}}.$

8. Find the domain of convergence of a functional series.

a) $\sum_{n=1}^{\infty} e^{nx},$

b) $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{\pi x}{n}.$

9. Study the uniform convergence of the functional sequence f_n on the given set.

a) $f_n(x) = \cos \frac{\pi x^n}{2}, x \in [0, 0.99],$ b) $f_n(x) = \operatorname{arctg}\left(e^{\frac{x}{n}}\right), x > 0.$

10. Study the uniform convergence of the functional series on the given set.

a) $\sum_{n=1}^{\infty} \frac{1}{(x+n)(x+n+1)}, x \geq 0,$ b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} x}{1+n^4 x^2}, x \in \mathbb{R}.$