

Deep Learning

Vazgen Mikayelyan

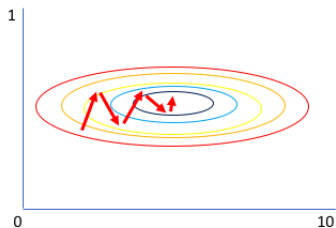
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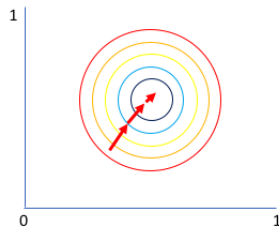
- 1 Data Normalization
- 2 Random Initialization
- 3 Dropout

Data Normalization

Why normalize?



Gradient of larger parameter dominates the update



Both parameters can be updated in equal proportions

Standard Normalization

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Denote $\mu^j = \frac{1}{n} \sum_{i=1}^n x_i^j$, $\sigma^j = \sqrt{\frac{1}{n} \sum_{i=1}^n (x_i^j - \mu^j)^2}$

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Our new data will be $(z_i, y_i)_{i=1}^n$, $z_i \in \mathbb{R}^k$, $y_i \in \mathbb{R}^m$.

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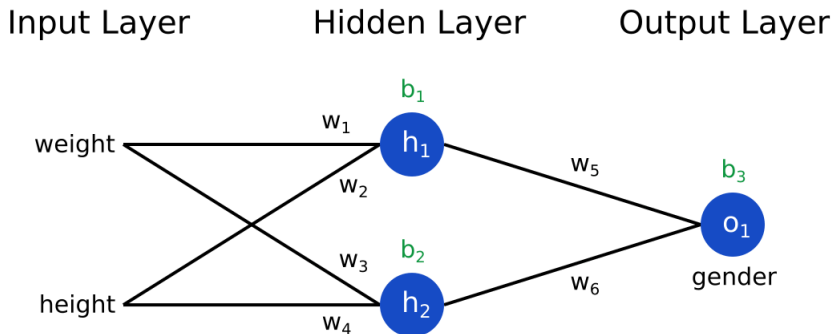
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$$w_i^1 = w_i^0 - \alpha \frac{\partial L}{\partial w_i}(w^0), \quad i = 1, \dots, 6,$$

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So we see that $w_1^1 = w_3^1 = \text{const}$, $w_2^1 = w_4^1 = \text{const}$.

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So we see that $w_1^1 = w_3^1 = \text{const}$, $w_2^1 = w_4^1 = \text{const}$. In the same way we can prove that $b_1^1 = b_2^1 = \text{const}$.

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- If the weights in a network start too small, then the signal shrinks as it passes through each layer until it's too tiny to be useful.
- If the weights in a network start too large, then the signal grows as it passes through each layer until it's too massive to be useful.

What's Xavier initialization?

Initializing the weights in your network by drawing them from a distribution with zero mean and a specific variance

$$\text{Var}(W) = \frac{1}{n_{in}},$$

where W is the initialization distribution for the neuron in question, and n_{in} is the number of neurons feeding into it. The distribution used is typically Gaussian or uniform.

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$$\text{Var}(W) = \frac{2}{n_{in} + n_{out}},$$

where n_{out} is the number of neurons the result is fed to.

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It easy to see that for all $i = 1, \dots, n$ we have

$$\text{Var}(W_iX_i) = E(X_i)^2 \text{Var}(W_i) + E(W_i)^2 \text{Var}(X_i) + \text{Var}(W_i) \text{Var}(X_i)$$

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to keep the variance of the input gradient the output gradient the same. These two constraints can only be satisfied simultaneously if $n_{in} = n_{out}$, so as a compromise, Glorot & Bengio take the average of the two:

$$\text{Var}(W_1) = \frac{2}{n_{in} + n_{out}}$$

Case of the activation function

Note that this was about a linear neuron, but it works also for tanh and sigmoid functions. For rectifying nonlinearities this is not true and there is a paper where authors suggest using

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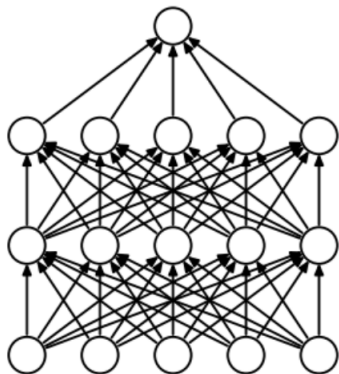
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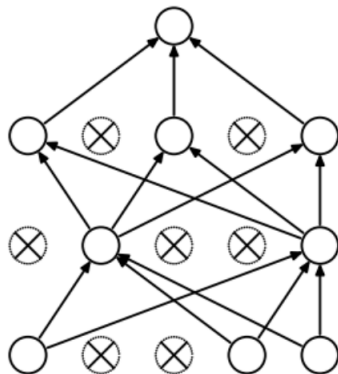
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(a) Standard Neural Net



(b) After applying dropout.