

Mathematics for Machine Learning: Homework 1

Deadline is 23.07.2020

July 16, 2020

1. Let

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 5 \end{bmatrix}, B = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 2 & 3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, D = \begin{bmatrix} 0 & -3 \\ 2 & 1 \end{bmatrix}.$$

Compute the indicated matrices.

- a) $AB + C^T$, c) $DA - AD$,
b) $B^T C^T + (CB)^T$, d) $(I_2 - D)^2$.

2. Use elementary row operations to reduce given matrix to row echelon form.

a) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, c) $\begin{bmatrix} 1 & 2 & -4 & -4 & 5 \\ 2 & 4 & 0 & 0 & 2 \\ 2 & 3 & 2 & 1 & 5 \\ -1 & 1 & 3 & 6 & 5 \end{bmatrix}$.

b) $\begin{bmatrix} 2 & -4 & -2 & 6 \\ 3 & 1 & 6 & 6 \end{bmatrix}$,

3. Solve the given system of equations using either Gaussian or Gauss-Jordan elimination.

a) $\begin{cases} x_1 + 2x_2 - 3x_3 = 9 \\ 2x_1 - 2x_2 + x_3 = 0 \\ 4x_1 - x_2 + x_3 = 4, \end{cases}$ c) $\begin{cases} \frac{1}{2}x_1 + x_2 - x_3 - 6x_4 = 2 \\ \frac{1}{6}x_1 + \frac{1}{2}x_2 - 3x_4 + x_5 = -1 \\ \frac{1}{3}x_1 - 2x_3 - 4x_5 = 8, \end{cases}$

b) $\begin{cases} 2x_1 + 3x_2 - x_3 + 4x_4 = 1 \\ 3x_1 - x_2 + x_4 = 1 \\ 3x_1 - 4x_2 + x_3 - x_4 = 2, \end{cases}$ d) $\begin{cases} \sqrt{2}x_1 + x_2 + 2x_3 = 1 \\ \sqrt{2}x_2 - 3x_3 = -\sqrt{2} \\ -x_2 + \sqrt{2}x_3 = 1. \end{cases}$

4. Check whether the given matrix is invertible and if it is, use the Gauss-Jordan method to find the inverse.

a) $\begin{bmatrix} 1 & a \\ -a & 1 \end{bmatrix},$

b) $\begin{bmatrix} 2 & 3 & 0 \\ 1 & -2 & -1 \\ 2 & 0 & -1 \end{bmatrix},$

c) $\begin{bmatrix} 0 & -1 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 1 & -1 & 3 & 0 \\ 0 & 1 & 1 & -1 \end{bmatrix}.$

5. Which of the following sets are subspaces of \mathbb{R}^3 ?

a) $A = \{(\lambda, \lambda + \mu^3, \lambda - \mu^3) : \lambda, \mu \in \mathbb{R}\},$

b) $B = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 - 2x_2 + 3x_3 = 1\},$

c) $C = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_2 \in \mathbb{Z}\}.$